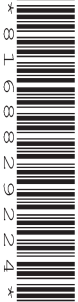


Wednesday 07 October 2020 – Afternoon

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

Time allowed: 2 hours



You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ ms}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **12** pages.

ADVICE

- Read each question carefully before you start your answer.

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient Rule } y = \frac{u}{v}, \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$$\sin \theta \approx \theta, \cos \theta \approx 1 - \frac{1}{2}\theta^2, \tan \theta \approx \theta \text{ where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi \right)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^n C_r p^r q^{n-r}$ where $q = 1 - p$

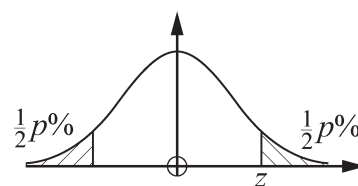
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Answer **all** the questions.

Section A (22 marks)

1 Simplify $\left(\frac{27}{x^9}\right)^{\frac{2}{3}} \times \left(\frac{x^4}{9}\right)$.

2 Express $\frac{a + \sqrt{2}}{3 - \sqrt{2}}$ in the form $p + q\sqrt{2}$, giving p and q in terms of a . [3]

3 The points A and B have position vectors $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 4 \\ 8 \end{pmatrix}$ respectively.

Show that the exact value of the distance AB is $\sqrt{101}$. [3]

4 Find the second derivative of $(x^2 + 5)^4$, giving your answer in factorised form. [5]

5 A child is running up and down a path. A simplified model of the child's motion is as follows:

- he first runs north for 5 s at 4 m s^{-1} ;
- he then suddenly stops and waits for 8 s;
- finally he runs in the opposite direction for 7 s at 3.5 m s^{-1} .

(a) Taking north to be the positive direction, sketch a velocity-time graph for this model of the child's motion. [2]

Using this model,

(b) calculate the total distance travelled by the child, [2]

(c) find his final displacement from his original position. [1]

- 6 A uniform ruler AB has mass 28 g and length 30 cm. As shown in Fig. 6, the ruler is placed on a horizontal table so that it overhangs a point C at the edge of the table by 25 cm.

A downward force of F N is applied at A. This force just holds the ruler in equilibrium. The contact force between the table and the ruler acts through C.



Fig. 6

- (a) Complete the force diagram in the Printed Answer Booklet, labelling the forces and all relevant distances. [2]
- (b) Calculate the value of F . [2]

Answer **all** the questions.

Section B (78 marks)

7 In this question you must show detailed reasoning.

The function $f(x)$ is defined by $f(x) = x^3 + x^2 - 8x - 12$ for all values of x .

- (a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$. [2]
- (b) Solve the equation $f(x) = 0$. [4]

8 Fig. 8.1 shows the cross-section of a straight driveway 4 m wide made from tarmac.

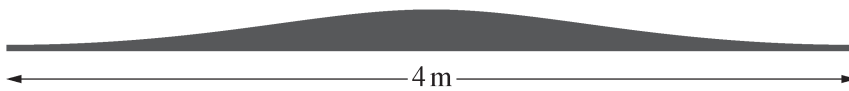


Fig. 8.1

The height h m of the cross-section at a displacement x m from the middle is modelled by $h = \frac{0.2}{1+x^2}$ for $-2 \leq x \leq 2$.

A lower bound of 0.3615 m^2 is found for the area of the cross-section using rectangles as shown in Fig. 8.2.

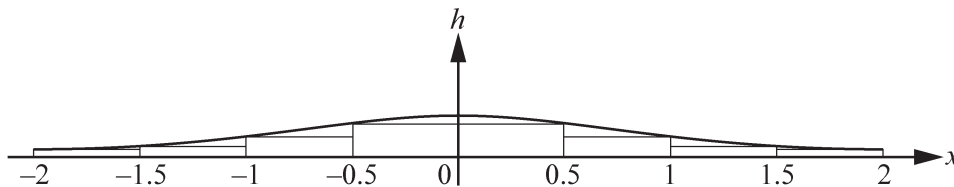


Fig. 8.2

- (a) Use a similar method to find an upper bound for the area of the cross-section. [3]
- (b) Use the trapezium rule with 4 strips to estimate $\int_0^2 \frac{0.2}{1+x^2} dx$. [2]
- (c) The driveway is 10 m long. Use your answer in part (b) to find an estimate of the volume of tarmac needed to make the driveway. [2]

- 9 A particle is moving in a straight line. The acceleration $a \text{ m s}^{-2}$ of the particle at $a = 0.8t + 0.5$. The initial velocity of the particle is 3 m s^{-1} in the positive x -direction. Determine whether the particle is ever stationary.

10 In this question you must show detailed reasoning.

Fig. 10 shows the curve given parametrically by the equations $x = \frac{1}{t^2}$, $y = \frac{1}{t^3} - \frac{1}{t}$, for $t > 0$.

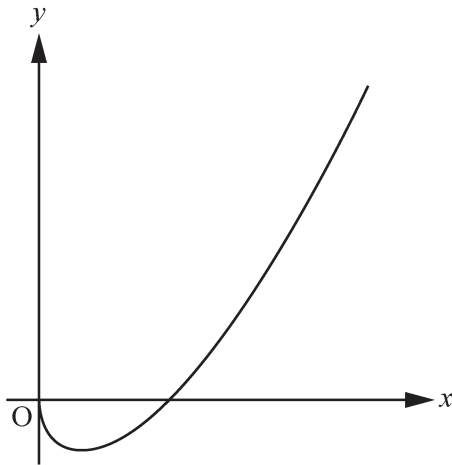


Fig. 10

- (a) Show that $\frac{dy}{dx} = \frac{3-t^2}{2t}$. [3]
- (b) Find the coordinates of the point on the curve at which the tangent to the curve is parallel to the line $4y + x = 1$. [3]
- (c) Find the cartesian equation of the curve. Give your answer in factorised form. [3]
- 11 A block of mass 2 kg is placed on a rough horizontal table. A light inextensible string attached to the block passes over a smooth pulley attached to the edge of the table. The other end of the string is attached to a sphere of mass 0.8 kg which hangs freely.
- The part of the string between the block and the pulley is horizontal. The coefficient of friction between the table and the block is 0.35 . The system is released from rest.
- (a) Draw a force diagram showing all the forces on the block and the sphere. [3]
- (b) Write down the equations of motion for the block and the sphere. [2]
- (c) Show that the acceleration of the system is 0.35 m s^{-2} . [4]
- (d) Calculate the time for the block to slide the first 0.5 m . Assume the block does not reach the pulley. [2]

- 12 A function is defined by $f(x) = x^3 - x$.
- (a) By considering $\frac{f(x+h) - f(x)}{h}$, show from first principles that $f'(x) = 3x^2$.
- (b) Sketch the gradient function $f'(x)$.
- (c) Show that the curve $y = f(x)$ has a single point of inflection which is not a stationary point. [3]

- 13 A projectile is fired from ground level at 35 m s^{-1} at an angle of θ° above the horizontal.
- (a) State a modelling assumption that is used in the standard projectile model. [1]
- (b) Find the cartesian equation of the trajectory of the projectile. [4]

The projectile travels above horizontal ground towards a wall that is 110 m away from the point of projection and 5 m high. The projectile reaches a maximum height of 22.5 m.

- (c) Determine whether the projectile hits the wall. [6]
- 14 Douglas wants to construct a model for the height of the tide in Liverpool during the day, using a cosine graph to represent the way the height changes.

He knows that the first high tide of the day measures 8.55 m and the first low tide of the day measures 1.75 m.

Douglas uses t for time and h for the height of the tide in metres. With his graph-drawing software set to degrees, he begins by drawing the graph of $h = 5.15 + 3.4 \cos t$.

- (a) Verify that this equation gives the correct values of h for the high and low tide. [1]

Douglas also knows that the first high tide of the day occurs at 1 am and the first low tide occurs at 7.20 am. He wants t to represent the time in hours after midnight, so he modifies his equation to $h = 5.15 + 3.4 \cos(at + b)$.

- (b) (i) Show that Douglas's modified equation gives the first high tide of the day occurring at the correct time if $a + b = 0$. [1]

- (ii) Use the time of the first low tide of the day to form a second equation relating a and b . [1]

- (iii) Hence show that $a = 28.42$ correct to 2 decimal places. [2]

- (c) Douglas can only sail his boat when the height of the tide is at least 3 m.
- Use the model to predict the range of times that morning when he cannot sail. [3]

- (d) The next high tide occurs at 12.59 pm when the height of the tide is 8.91 m.
- Comment on the suitability of Douglas's model. [2]

- 15 Fig. 15 shows a particle of mass m kg on a smooth plane inclined at 30° to the horizontal. Vectors \mathbf{i} and \mathbf{j} are parallel and perpendicular to the plane, in the directions shown.

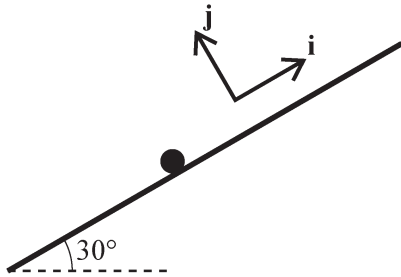


Fig. 15

- (a) Express the weight \mathbf{W} of the particle in terms of m , g , \mathbf{i} and \mathbf{j} . [2]

The particle is held in equilibrium by a force \mathbf{F} , and the normal reaction of the plane on the particle is denoted by \mathbf{R} . The units for both \mathbf{F} and \mathbf{R} are newtons.

- (b) Write down an equation relating \mathbf{W} , \mathbf{R} and \mathbf{F} . [1]
- (c) Given that $\mathbf{F} = 6\mathbf{i} + 8\mathbf{j}$,
- show that $m = 1.22$ correct to 3 significant figures,
 - find the magnitude of \mathbf{R} . [6]

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